

The Breaking Point

Using Rational Functions to Solve Problems

7.5

LEARNING GOALS

In this lesson, you will:

- Model situations with rational functions.
- Use rational expressions to solve real-world problems.

In Psychology, the term *breaking point* refers to a traumatic moment when a person breaks down. This may happen because of emotional or physical stress, and often ends up being the point at which there must be some sort of resolution to a problem.

Television and movies regularly use this as the moment of climax or resolution to their stories. A criminal may break down because of intense feelings of guilt during a police interrogation. Two characters in a story may try to avoid a conflict over something until their bad feelings reach a breaking point where they must somehow resolve their conflict. During an intense sports competition, often one team struggles to win while the other team hits a breaking point that is too much to overcome.

Can you think of any specific movies where a character reaches a breaking point? How was this moment used in the story?

PROBLEM 1 Start Applying Yourself, Rational Function!

Recall that a rational expression is the ratio of two polynomials. Rational expressions can be used to solve problems that involve comparing two quantities of the same unit of measure.



1. Dellilah is making her own salad dressing out of red vinegar and olive oil. It's a new recipe so she has to determine the correct proportions. She mixes 10 teaspoons of vinegar and 16 teaspoons of olive oil. After she stirs the mixture, she realizes it's not the consistency she wants, so she adds more olive oil.
 - a. What is the ratio of red vinegar to olive oil if she adds 6 teaspoons more of olive oil?
 - b. What is the ratio of red vinegar to olive oil if she adds 10 teaspoons more of olive oil?
 - c. Write an expression to represent the ratio of red vinegar to olive oil. Let x represent the number of additional teaspoons of olive oil added to the recipe.
 - d. Describe the behavior of the ratio as the number of additional teaspoons of olive oil increases. Show all of your work and explain your reasoning.
 - e. The recommended ratio of vinegar to olive oil is 1:7. Determine the amount of olive oil that she must add to the mixture. Show all of your work and explain your reasoning.

Remember to use your proportional reasoning skills.



- f. What are the domain and range of the function? Explain your reasoning.



2. A door-to-door salesperson for TV Bonanza Cable Company offers cable television for only \$55.95 per month. However, there is a one-time installation cost of \$180.

- a. Determine the total cost of cable for the first two months? What is the average cost per month over the first two months?

Don't forget about the upfront charges.



- b. Determine the total cost of cable for the first year. What is the average cost per month over the first year?

c. Write an equation to represent the total cost of cable for x months.

d. Write an equation to represent the average cost of cable for x months.

e. A competitor offers a similar product for \$65 per month and no installation charges. Who is offering the better deal? Show all work and explain your reasoning.

3. Tracy and Adrian model the following problem with a rational function.

Crunchy College Kid Snack Company manufactures a new brand of trail mix containing peanuts, almonds, and chocolate. Each package contains 400 grams of trail mix, with 50% peanuts, 35% almonds, and 15% chocolate. Herbert loves chocolate. When he gets the bag home, he wants to add enough chocolate so that the mixture is 50% chocolate. How many grams of chocolate should he add?

Tracy

I must first determine the amount of chocolate in the bag.

$$(0.15) \cdot (400) = 60\text{g}$$

The ratio of chocolate to total trail mix must increase to 50%. Adding chocolate increases the total amount of trail mix, so the new ratio is $\frac{60+x}{400+x}$. I can set up the proportion $\frac{60+x}{400+x} = 0.50$. The x-value represents the amount of additional chocolate.

Adrian

The current chocolate to trail mix ratio is $\frac{15}{100}$. Adding chocolate to get a mixture add lead above to clear crash with fraction as well as the total, so the rational equation becomes $\frac{15+x}{100+x} = 0.50$. The x-value of the intersection point represents the amount of chocolate Herbert must add.

- a. Who is correct? Explain your reasoning. If necessary, include the error in the student's reasoning.

- b. Determine the grams of chocolate that Herbert must add to the trail mix to get a mixture that is 50% chocolate.

Use proportional reasoning to solve each equation.



4. A common misconception is that you can determine how far away a storm is by measuring the time between thunder and lightning. In reality, though, the time between seeing lightning and hearing thunder is a function of both distance and temperature. The time between seeing lightning and hearing thunder is represented by the function $Time = \frac{d}{1.09t + 1050}$, where d is the distance (feet) between the observer and the lightning, and t is the temperature (Fahrenheit).

- a. If the temperature outside is 70 degrees and you count 3 seconds between the thunder and the lightning, approximately how far away is the storm? Show all of your work and explain your reasoning.

- b. If the temperature is 80 degrees and you estimate half a second between thunder and lightning, how far away is the storm? Show all of your work and explain your reasoning.



- c. On a 60 degree day, what is the time between thunder and lightning when the storm is directly overhead? Show all work and explain your reasoning.



PROBLEM 2 For The Record


1. In football, a quarterback's completion percentage is the ratio of the number of complete passes to the total number of pass attempts. The current record holder for highest completion percentage is Chad Pennington who completed 66% of his passes over the course of his career in the National Football League. The quarterback in second place completed 3843 passes out of 5853 attempts. Estimate the number of consecutive complete passes the second place quarterback must throw in order to break the record. Show all of your work and explain your reasoning

2. Josie compares two different refrigerators at the local hardware store. The sales tags are shown.

ICY COLD

- Crushed Ice Dispenser
- Price \$699.00
- Uses 75 kilowatt hours of electricity per month

COOL AS A CUCUMBER

- Gold Star EPA efficiency rating
- Price: \$825
- Uses 30 kilowatt hours of electricity per month

Josie does some research online and learns that a kilowatt hour costs approximately \$0.06. She also learns that the average refrigerator lasts about 10 years.

- a. Write a function to represent the average cost of each refrigerator per month.
- b. Which refrigerator will have a lower average monthly cost over the next ten years? Show all of your work and explain your reasoning.

6. The ancient Greeks felt as though certain rectangles in art and architecture were much more pleasing to the eye than others. When the ratio of the sum of the length and width to the length is approximately 1.618, they felt the rectangle was perfectly proportionate. This ratio came to be known as the *Golden Ratio*. Determine the length and width of several rectangles with dimensions that are in the *Golden Ratio*.

Many buildings today are constructed with windows that are in the golden ratio. In studies, people today still find rectangles with dimensions in this ratio most pleasing to look at.



Be prepared to share your solutions and methods.